

# Системные сетевые риски межбанковского рынка России

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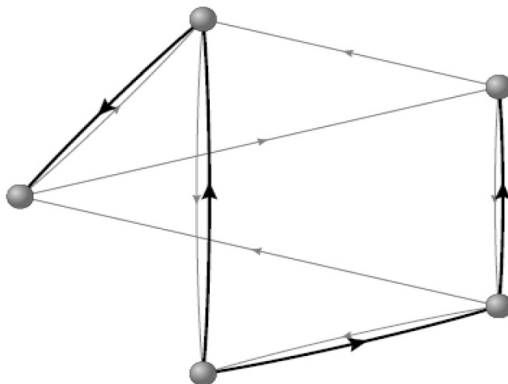
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# BOE view of an interbank network

P. Gai, S. Kapadia, "Contagion in financial networks"

Bank of England Working Paper No.383



## Assets

## Liabilities

Interbank credits	Capital
	Interbank liabilities
Illiquid assets	Deposits

P. Gai, S. Kapadia, "Contagion in financial networks"

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- Solvency condition for a bank  $i$ :

$$(1 - \phi)A_i^{\text{IB}} + qA_i^{\text{M}} - L_i^{\text{IB}} - D_i > 0$$

- Definition of variables:

- $A^{\text{IB}}$ : liquid interbank assets
- $A^{\text{M}}$ : illiquid assets
- $L^{\text{IB}}$ : interbank liabilities
- $\phi$ : Fraction of contacts going bankrupt
- $D$ : Deposits
- $K = A^{\text{IB}} + A^{\text{M}} - L^{\text{IB}} - D$ : capital buffer

# INTERBANK NETWORKS: CONTAGION MODELLING

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- Vulnerable banks are those that default because of a default of one of their counterparties (i.e.  $\phi = 1/j$ )
- A bank  $i$  with  $j$  incoming links is thus vulnerable in this sense if

$$K_i - (1 - q)A_i^M < \frac{A_i^{IB}}{j}$$

- Because of the randomness of the capital buffer  $K_i$  (e.g. due to that of the deposits) contagion is probabilistic with a probability  $v_j$ :

$$v_j = \text{Prob} \left[ K_i - (1 - q)A_i^M < \frac{A_i^{IB}}{j} \right]$$

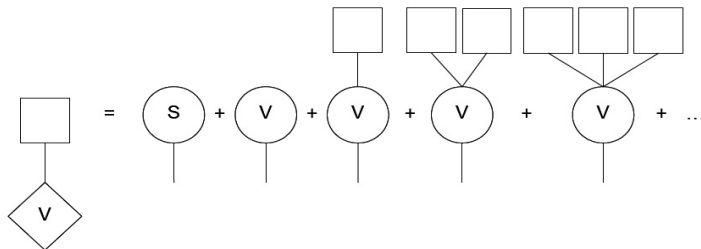
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- $H_1(y)$  - generating function for the probability of reaching an outgoing vulnerable cluster of given size by following a random outgoing link from a vulnerable bank

$$H_1(y) = \Pr[\text{reach safe bank}] + y \sum_{j,l} v_j \cdot r_{jk} [H_1(y)]^k$$



- Generating function for the size of a vulnerable cluster

$$H_0(y) = 1 - G_0(1) + yG_0[H_1(y)]$$

- Size of the vulnerable cluster  $S$ :

$$S = G_0(1) + \frac{G'_0(1)G_1(1)}{1 - G'_1(1)}$$

- Phase transition:

$$G'_1(1) = 1 \quad \Leftrightarrow \quad \sum_{j,k} j \cdot k \cdot v_j \cdot p_{jk} = z$$

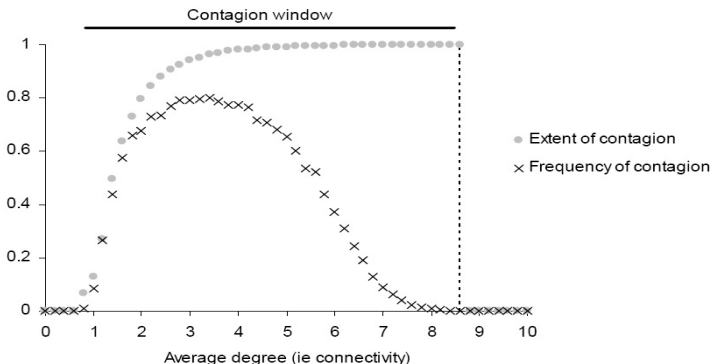
- In this case: opposing effects from  $j \cdot k$  and  $v_j$  lead to two phase transitions and a contagion window in  $z$ !

# INTERBANK NETWORKS: CONTAGION MODELLING

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- Benchmark case. Contagion of more than 5 % of the network resulting from the default of a single bank



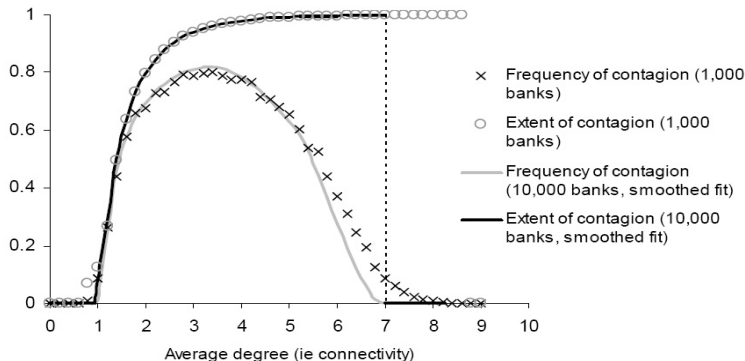


# INTERBANK NETWORKS: CONTAGION MODELLING

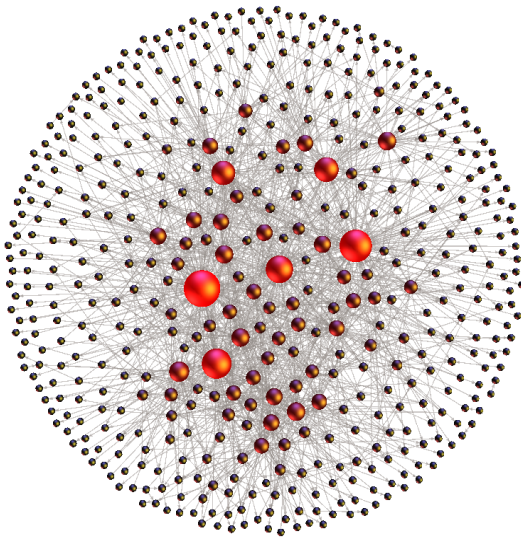
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## • Comparison with analytic calculation



# Russian interbank market



# Russian interbank network: data description

- Uncollateralized interbank rouble deposits of all maturities in the period from January 11, 2011 till December 30, 2011 are considered.
- Interbank network for  $N$  banks is fully characterized by an oriented weighted graph  $G^W = (N, W)$ , where  $W = \{w_{ij}\}$  is an  $N \times N$  matrix of  $w_{ij} > 0$  of liabilities of the bank  $i$  with respect to the bank  $j$ .
- By definition the outgoing links correspond to liabilities, the incoming ones - to claims.
- The interbank network graph is scale-free in both in- and out- degrees and is characterized by significant clustering.

# Definition of vulnerability

- Solvency coefficient  $H1$  as defined by CBRF:

$$H1 = \frac{K}{\sum_i A_i K p_i + PP + OP + others}$$

- Here

- $K$  is capital
- $K p_i$  - risk coefficients
- All instruments are divided into 5 groups  $i = 1, \dots, 5$  and  $K p_1 = 0$ ,  $K p_2 = 20 \%$ , etc. For the interbank market the risk coefficient is 20 %
- $PP$  - market risk
- $OP$  - operational risk
- others - other contributions

# Definition of vulnerability

- A default condition is

$$H1 = \frac{K}{\sum_i A_i K p_i + PP + OP + others} < H1^*$$

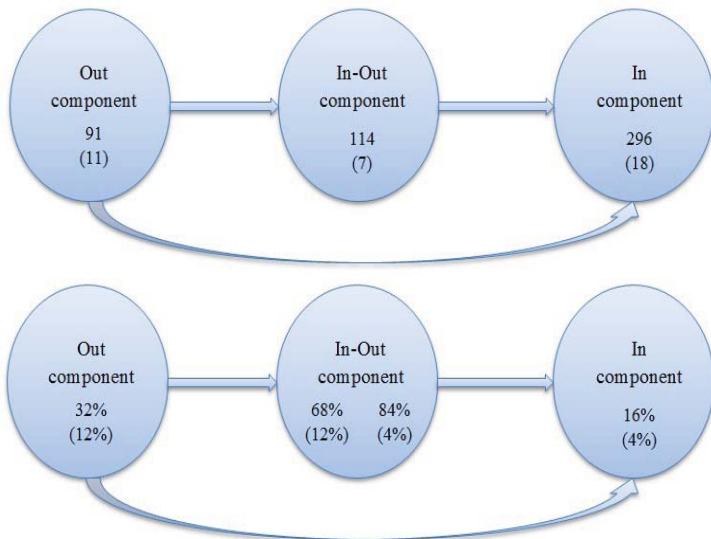
where for banks  $H1^* = 10 \%$ , for others -  $H1^* = 12 \%$

- Calculation using  $H1$ :

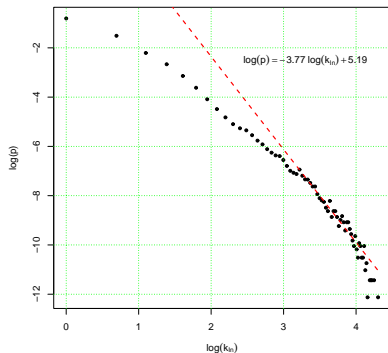
$$H1 \Rightarrow \frac{K - P}{\sum_i A_i K p_i + PP + OP + others}$$

where  $P$  is a reserve kept for the case when one or several counteragents default.

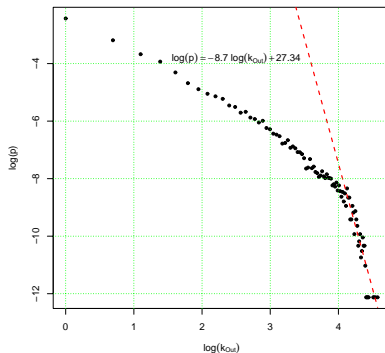
# Interbank network: bow-tie structure, nodes and weights



# In- and Out- degree distributions

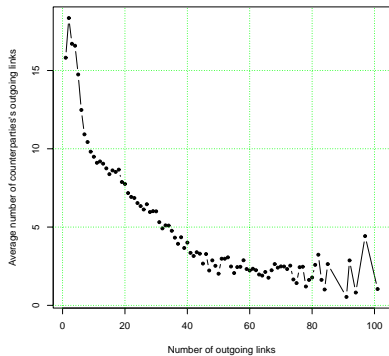
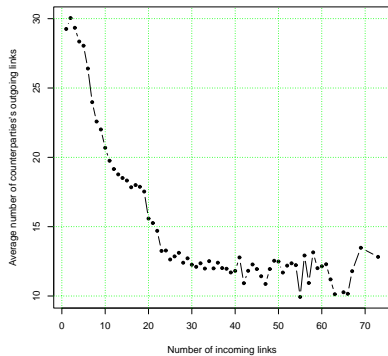


$$P(k_{in}) \sim \frac{1}{k_{in}^{3.77}}$$



$$P(k_{out}) \sim \frac{1}{k_{out}^{8.7}}$$

# Disassortativity

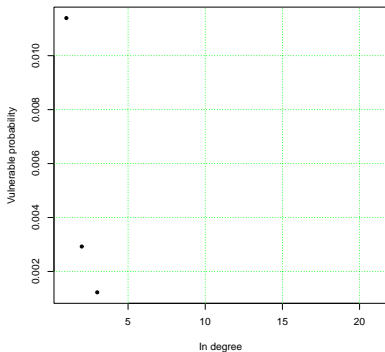




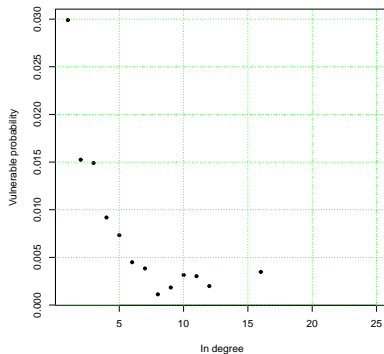
# Empirical default distributions

Probability that at least one incoming link is vulnerable:

Out  $\rightarrow$  In-Out



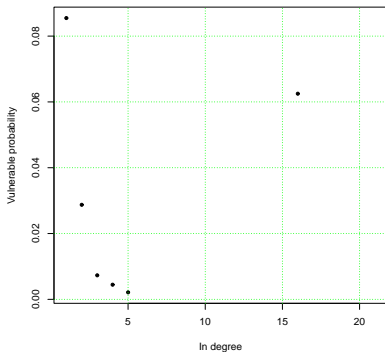
In-Out  $\rightarrow$  In-Out



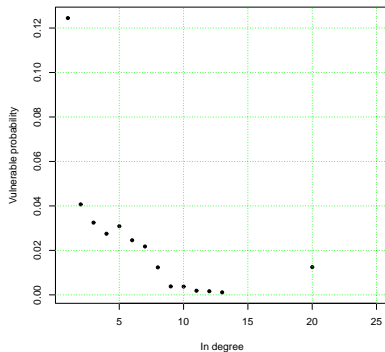
# Empirical default distributions

Probability that at least one incoming link is vulnerable:

Out  $\rightarrow$  In

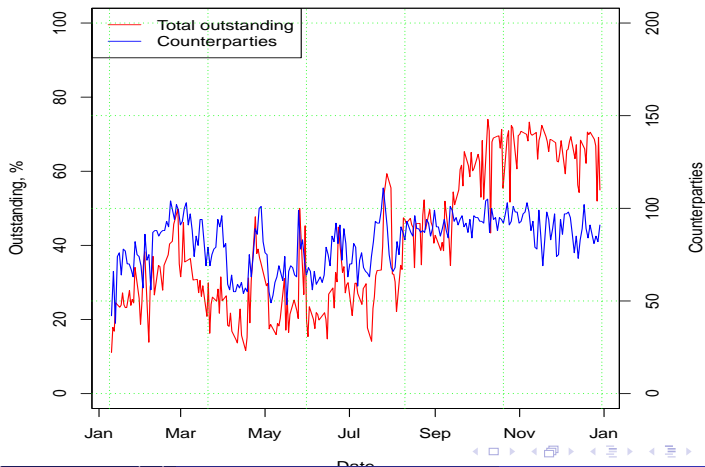


In-Out  $\rightarrow$  In

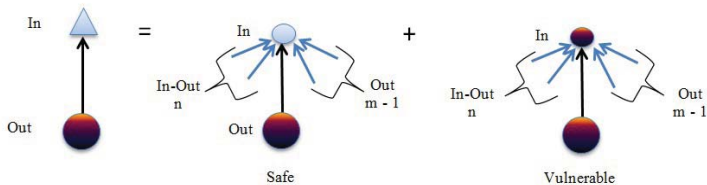


# Strongly connected component

- There exists a strongly connected component
- The weight of this component did significantly increase

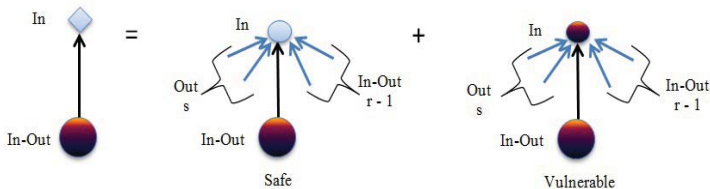


# Contagion tree Out $\rightarrow$ In



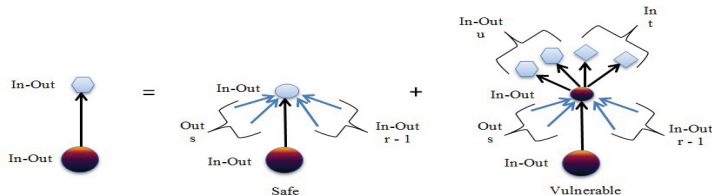
$$L_{i,j}(y) = \sum_{n,m}^{\infty} P_{\text{Out}/\text{In}}(n, m|i, j) \left[ (1 - v_m^{\text{Out}/\text{In}}) + v_m^{\text{Out}/\text{In}} y \right]$$

# Contagion tree In-Out $\rightarrow$ In



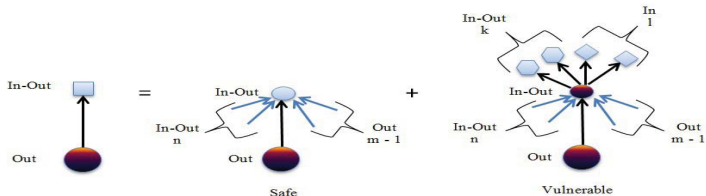
$$N_{k,l}(y) = \sum_{s,r}^{\infty} P_{In-Out/In}(s, r|k, l) \left[ (1 - v_r^{In-Out/In}) + v_r^{In-Out/In} y \right]$$

# Contagion tree In-Out $\rightarrow$ In-Out & In



$$M_{k,l}(x, N(y)) = \sum_{u,t,s,r}^{\infty} P_{In-Out/In-Out}(u, t, s, r | k, l) \\ * \left[ (1 - v_r^{In-Out/In-Out}) + x v_r^{In-Out/In-Out} M_{u,t}^u(x, N(y)) N_{u,t}^t(y) \right]$$

# Contagion tree Out $\rightarrow$ In-Out $\rightarrow$ In-Out & In



$$K_{i,j}(x, y) = \sum_{k,l,n,m}^{\infty} P_{Out/In-Out}(k, l, n, m|i, j) \\ * \left[ (1 - v_m^{Out/In-Out}) + x v_m^{Out/In-Out} [M_{k,l}(x, y)]^k [N_{k,l}(y)]^l \right]$$

# Contagion clusters In-Out $\rightarrow$ In-Out & In

- Let  $F(x, y)$  be the generation function for the probability for a bank from In-Out being linked with In-Out and In components:

$$F(x, y) = \sum_{i,j=0}^{\infty} p_{ij}^{InOut} x^i y^j$$

- The generation function for default cluster originating in In-Out then reads:

$$\mathcal{G}_{InOut}(x, y) = F(M(x, N(y)), N(y))$$

- The average size of default clusters is given by

$$\left. \frac{d\mathcal{G}_{InOut}(x, x)}{dx} \right|_{x=1} = 1$$



# Contagion clusters Out $\rightarrow$ In-Out & In

- Let  $G(x, y)$  be the generation function for the probability for a bank from Out being linked with In-Out and In components:

$$G(x, y) = \sum_{i,j=0}^{\infty} p_{ij} x^i y^j$$

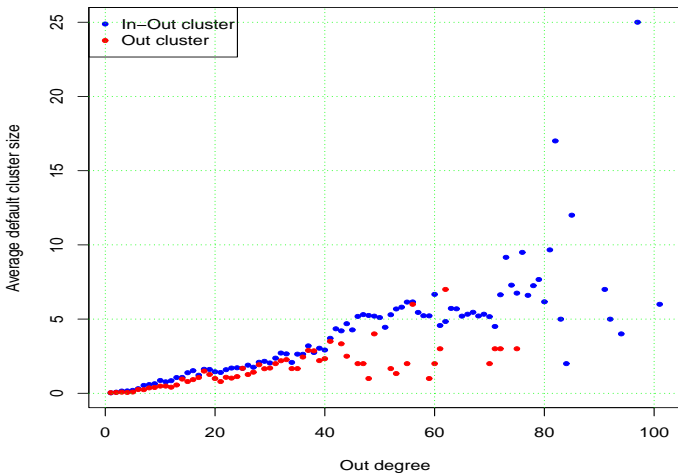
- The generation function for default cluster originating in Out then reads:

$$\mathcal{G}_{Out}(x, y) = G(K(M(x, N(y)), N(y)), L(y))$$

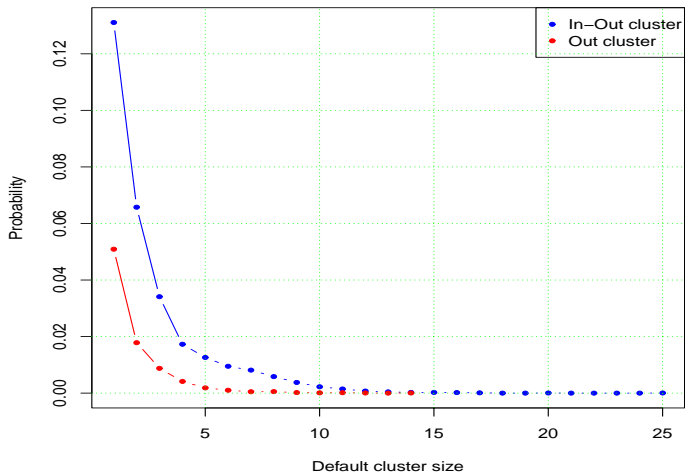
- The average size of default clusters is given by

$$\left. \frac{d\mathcal{G}_{Out}(x, x)}{dx} \right|_{x=1} = 1$$

# Simulation: dependence upon out-degree



# Simulation: default cluster size distribution



- Taking into account the bow-tie structure of the interbank network is very essential.
- Despite of the complicated topology of the original graph, the default clusters are (almost) always tree-like.
- This allows to describe default clusters in terms of generating functions taking into account the bow-tie structure of the original interbank network graph.
- The realistic contagion in the RF interbank market is a relatively small effect, nothing dramatic.